

The *HST* large programme on NGC 6752 – IV. The White Dwarf sequence

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ABSTRACT

We present our final study of the white dwarf cooling sequence (WD CS) in the globular cluster NGC 6752. The investigation is the main goal of a dedicated *Hubble Space Telescope* large program, for which all the observations are now collected. The WD CS luminosity function (LF) is confirmed to peak at $m_{F606W} \simeq 29.3 \pm 0.1$, consistent within uncertainties with what has been previously reported, and is now complete down to $m_{F606W} \simeq 29.7$. We have performed robust and conclusive comparisons with model predictions that show how the theoretical LF for hydrogen envelope WD models closely follow the shape of the empirical LF. The magnitude of the peak of the observed LF is matched with ages between 12.7 and 13.5 Gyr, consistent with the cluster age derived from the main sequence turn off and subgiant branch. We also find that the impact of multiple populations within the cluster on the WD LF for m_{F606W} below 27.3 is negligible, and that the presence of a small fraction of helium envelope objects is consistent with the data. Our analysis reveals a possible hint of an underestimate of the cooling time-scales of models in the magnitude range $28.1 < m_{F606W} < 28.9$. Finally, we find that hydrogen envelope models calculated with a new tabulation of electron conduction opacities in the transition between moderate and strong degeneracy provide WD ages that are too small in comparison to the main sequence turnoff age.

Key words: white dwarfs – globular clusters: individual: NGC 6752.

1 INTRODUCTION

Located at a distance of about 4 kpc (Baumgardt & Vasiliev 2021), NGC 6752 is one of the closest Milky Way globular clusters (GCs) to the Sun. With its proximity and low interstellar reddening, NGC 6752 was one of the first GCs to reveal the presence of white dwarfs (WDs; Richer 1978) before the advent of *Hubble Space Telescope* (*HST*). A small sample of bright WDs in NGC 6752 observed with *HST* was employed by Renzini et al. (1996) to determine the cluster distance with a technique analogous to the main sequence (MS) fitting. More recently, Bedin et al. (2019b) using deeper *HST* observations, obtained a first, preliminary, WD luminosity function (LF) that reached the end of the cooling sequence (CS) of hydrogen envelope WDs. Thus, NGC 6752 belongs to an exclusive club of GCs for which the end of the WD CS has been observed, including

NGC 6397 (Richer et al. 2006; Hansen et al. 2007), M 4 (Bedin et al. 2009), and 47 Tuc (Hansen et al. 2013).

One peculiarity of NGC 6752 is the atypical horizontal branch (HB) morphology for its metallicity $[Fe/H] \sim -1.6$, Gratton et al. (2005). There are no RR Lyrae stars in its HB, which has a pronounced extension to the blue, related to the presence of multiple populations with different initial helium abundances (e.g. Bastian & Lardo 2018; Cassisi et al. 2014, and references therein). This blue morphology affects the bright part of the WD CS, as shown very recently by Chen et al. (2022) based on near-ultraviolet (UV) imaging observations with *HST* Wide Field Camera 3. Using synthetic HB modelling by Cassisi et al. (2014) and theoretical results on WD progenitor evolution by Althaus et al. (2015), Chen et al. (2022) have shown that the bright part of the CS is populated by two types of WDs. A ‘slow’ – in terms of evolutionary speed – component supported by envelope hydrogen burning, and a ‘canonical’ component which is

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cooling without any substantial contribution from nuclear burning (more on this later).

This work is the fourth of a series of articles aimed at the scientific exploitation of an *HST* multicycle large program focused on the GC NGC 6752. The previous three publications were based on data from the first epoch of observations. Bedin et al. (2019a, Paper I) presented the discovery of a dwarf spheroidal galaxy that needs to be masked out in studies of the faint population of NGC 6752. Milone et al. (2019, Paper II) characterized the multiple stellar populations of NGC 6752; Finally, Bedin et al. (2019b, Paper III) reached, for the first time in NGC 6752, the end of the hydrogen envelope WD CS.

In this study, we have employed data for all epochs, which significantly improves the definition of the cluster WD CS and its LF, and allows us to perform a more robust and reliable investigation of the agreement between WD and main sequence turn off (TO) ages, the effect of the cluster multiple populations on the faint end of the CS, and the impact of helium envelope WDs on the LF. We have also been able to test two different treatments of the electron conduction opacities in the regime between moderate and strong degeneracy, which are crucial for the modelling of WD envelopes and have a major impact on the cooling times of the models (see Cassisi et al. 2021).

The article is organized as follows: Section 2 presents the observations, while section 3 gives details for the data reduction. Section 4 provides a brief description of the artificial star tests (ASTs). Section 5 describes the selection criteria used to obtain the cluster colour–magnitude diagram (CMD). Section 6 explains the decontamination of the cluster sample using proper motions (PMs). Section 7 presents the derived empirical WD LF. Section 8 compares the empirical LF with theory. Section 9 summarizes our results.

2 OBSERVATIONS

This study is based on images collected with the *Wide-Field Channel* (WFC) of the *Advanced Camera for Surveys* (ACS) onboard *HST* under the multicycle large program: ‘*The end of the White Dwarf Cooling Sequence of NGC 6752*’, programs: GO-15096 and GO-15491 (PI: Bedin). Paper III presented the first epoch of data obtained as part of program GO-15096, collected between 2018 September 7 and 18. These data consist of deep exposures of ~ 1270 s each, 19 taken with the F814W filter and 56 taken with the F606W filter. Short exposures of ~ 45 s each were also collected at the beginning of each orbit, 10 with the F814W filter and 27 with the F606W filter. Five of the planned 40 orbits failed (some only partially) because of poor guide-star acquisition and were repeated between 2019 August 1 and 15, resulting in an additional 5×45 s and 10×1270 s exposures with the F814W filter.

The second half of the data were collected between 2021 September 2 and 11 as a part of GO-15491. Due to changes in the *HST*-phase-II policies, deep exposures were on average shorter by ~ 55 s to allow for more ease in the scheduling. We obtained long exposures of ~ 1215 s each, 56 images with the F814W filter and 20 images with the F606W filter; and short exposures of ~ 45 s, each 12 with the F814W filter and 28 images with the F606W filter. Two orbits were lost due to poor guiding, and these observations were repeated on 2022 February 14; with two short (45 s) and four deep (1209 s) exposures with the F814W filter. All images were collected between ~ 2018.68 and ~ 2022.12 , resulting in four epochs over a time-baseline of ~ 3.5 yr. In a forthcoming publication, we use these multi-epoch observations to conduct an astrometric analysis of stars bright enough to be detected in individual images ($V \lesssim 28$) to determine the absolute motions, parallax, and internal velocity dispersion of the

cluster. Here, we analyse all of the images simultaneously to detect the faintest WD members of the cluster.

3 DATA REDUCTION

The data reduction was essentially identical to that presented in Paper III but with about twice the number of images. We refer the interested reader to Paper III and previous publications for details on the procedures, and provide here a brief description.

We downloaded from the MAST archive¹ the `f1c` images, which were pre-processed with *Space Telescope Science Institute (STScI)*’s pipeline. The `f1c` images are corrected for dark current, bias, flat-fielding, and charge-transfer efficiency (CTE) losses (following the Anderson & Bedin 2010 recipes for pixel-based correction) with the latest reference files, but with no resampling of the pixels.

We first conducted a ‘*first-pass*’ analysis to derive optimized point-spread functions (PSFs) for all images and establish a common distortion-free reference frame. Fluxes and positions for relatively bright (down to ~ 3.5 mag below saturation), unsaturated stars were extracted from each `f1c` image using software developed by J. Anderson, described in Anderson & King (2006). Each image was analysed separately to create a tailored PSF in order to account for the particular breathing state of *HST*’s telescope tube, which affects both spatial and temporal variations relative to the library PSFs provided by Anderson (2006). This tailoring of PSFs was done with prescriptions introduced in Anderson & Bedin (2017) for WFC3/UVIS, and later extended to ACS/WFC by Bellini, Anderson & Grogin (2018). Next, both positions and fluxes are corrected for the geometric distortion of the detector by Anderson (2006),² which affects pixel areas and hence fluxes. These geometric corrections were used to produce a common, distortion-free reference frame – based on cluster members – to which all individual images are linked. Note that during the ~ 3.5 yr between the first and last epoch, cluster members are expected to have internal motions on the order of ~ 1 mas; field objects will have much larger motions (more about motions in Section 6).

This ‘*first-pass*’ analysis yields a distortion-free reference frame, with positions accurate to milli-arcsec (mas) levels, and magnitudes zero-pointed to milli-magnitude (mmag) precision levels. With these calibrations in hand, we performed a ‘*second-pass*’ analysis in which all of the pixels from all of the images are analysed simultaneously to search for the faintest sources in the field, in particular those not detectable in individual images. This analysis was done using the most recent version of the code (KS2) developed by J. Anderson (first presented in Anderson et al. 2008), and applied in several other GC analyses (cf. Scalco et al. 2021 and references therein). The KS2 code goes through multiple iterations of finding, modelling, and subtracting point sources from the image, starting from the brightest sources and moving progressively to fainter sources in the subtraction residuals. The code solves for positions and fluxes, as well as other important diagnostic parameters such as the local sky noise (`rmsSKY`) which documents how noisy the investigated patch of sky is, and the `RADXS` parameter (introduced in Bedin et al. 2009) which documents how well the source flux distribution resembles that of the PSF. The `RADXS` parameter is the most efficient diagnostic to eliminate faint unresolved galaxies, poorly measured stars perturbed by non-modellable neighbors, cosmic ray hits, PSF substructure,

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²Publicly available at <https://www.stsci.edu/~jayander/HSTIPASS/LIB/GD/Cs/STDGCs/>.

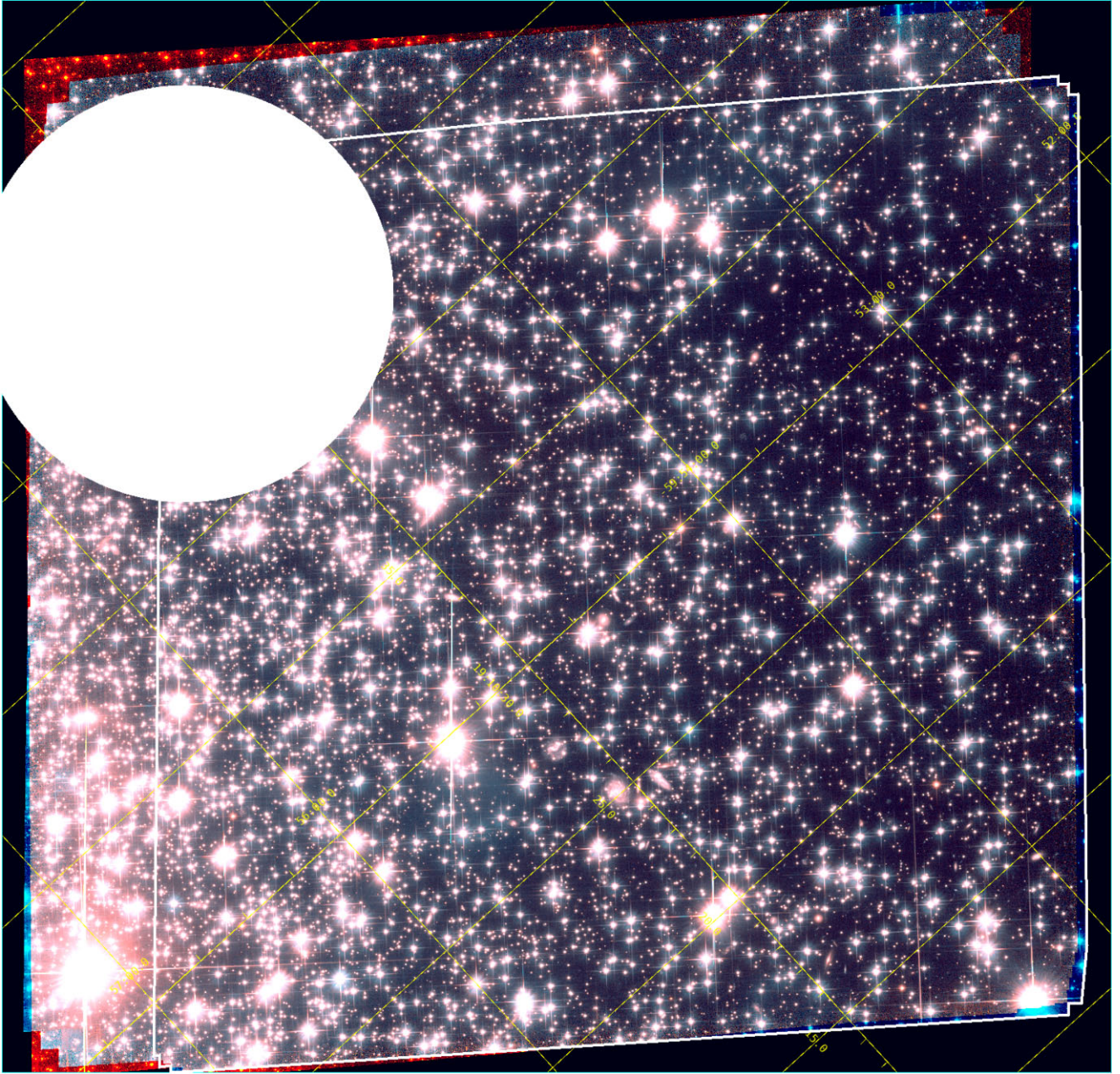


Figure 1. Tri-chromatic view of about 250×250 arcsec² for the entire ACS/WFC data set from *HST* programs GO-15096 & GO-15491. The white lines indicate the region observed at first epoch. The circular white region is rejected from this work, as it used to mask out the *Bedin I* dwarf spheroidal galaxy in background (Paper I). The yellow lines and labels indicate the *Gaia* DR3 reference frame.

diffraction spikes, and other artefacts. Stars that are saturated in the deep exposures have valid first-pass measurements from the short exposures and are linked to the deep-exposure-based master frame via common unsaturated stars.

Photometry was calibrated to the ACS/WFC Vega-mag system with the procedure given in Bedin et al. (2005) using encircled energy and zero points available from STScI.³ For the derived magnitudes in this photometric system, we adopt the symbols m_{F606W} and m_{F814W} .

The KS2 code also produces stacked images for visual examination, which we release as part of the electronic material associated

to this work. We combined these stacks for the two filters to produce a colour view of the studied region, shown in Fig. 1. A tri-chromatic view was obtained using F606W and F814W for the blue and red channels, while using a wavelength-weighted mean of F606W and F814W for the green channel. In the figure, we masked out a region centred on the dwarf spheroidal galaxy *Bedin I* (coordinates from Paper I) with a radius of 1000 pixels (~ 50 arcsec). This region is completely excluded from our analysis, as it proved impossible to achieve a useful discrimination in the CMD between the location of the dwarf galaxy’s stars and the WDs of NGC 6752.

The absolute astrometric registration to the International Celestial Reference System was achieved using sources in common with *Gaia* Early Data Release 3 (EDR3; Gaia Collaboration 2021). Tabulated

³www.stsci.edu/hst/acs/analysis/zeropoints

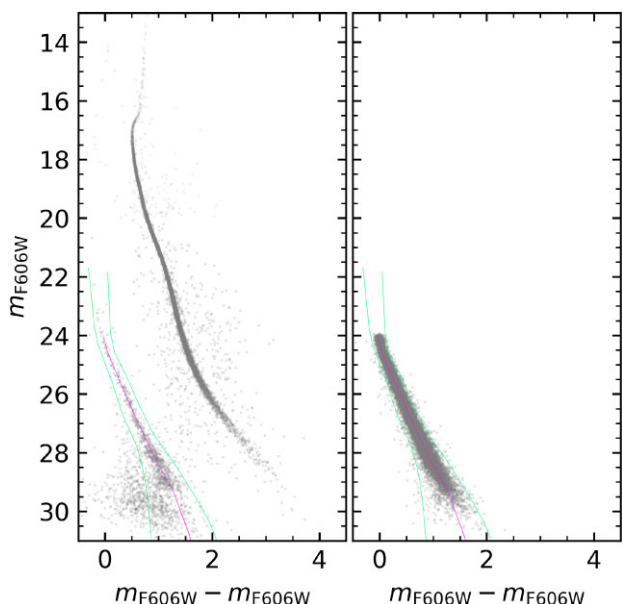


Figure 2. (*Left:*) Preliminary CMD for the sources within the studied field. Only sources with $\text{RADXS} < 0.1$ in both F606W and F814W are displayed. The line in magenta, obtained as described in Section 4, is the fiducial line for the WD CS of NGC 6752. The green lines define the region of the CMD within which we will count the WDs. (*Right:*) With the same scale, we show the CMD for artificial stars. The same magenta and green lines of the left-hand panel are also displayed here. As it can be seen, the lines in green define a region that is generous enough to include both the bulk of the observed real WDs in the left-hand panel and the artificial stars added along the WD fiducial line recovered with large photometric errors. Note that the exact location of the magenta line does not affect the number of WDs counted within the region defined by the two green lines.

PMs in *Gaia* EDR3 were transformed to the average Julian Day of images collected during the first epoch, following the procedures in Bedin & Fontanive (2018).

In the left-hand panel of Fig. 2, we show a preliminary CMD for sources having $|\text{RADXS}| < 0.1$ in both F606W and F814W. In the next section, we will employ this CMD to define the fiducial line of the WD CS of NGC 6752. This fiducial will be used to generate artificial stars along it, which in turn will enable us to define the region within which to count the WDs of NGC 6752, and to carefully select well-measured stars (see Section 5).

4 ARTIFICIAL STARS

When studying faint sources, ASTs have several key roles, specifically: (i) ASTs are used to track and correct for systematic errors between input magnitude and recovered magnitudes (see Bedin et al. 2009); (ii) ASTs are also employed to estimate the random errors and therefore define which sources have a position on the CMD consistent with being WDs; (iii) ASTs are used to check and define the selections on the distribution of diagnostic parameters, such as RADXS or rmsSKY ; and finally – and of fundamental importance – (iv) ASTs are used to assess the completeness of the sample.

In creating the ASTs, the first step is to choose where to add them in the CMD, and where to add them spatially across the field of view. Since we are studying the WD CS of NGC 6752, on the CMD we define a fiducial line, drawn by hand, along the bulk of the observed WDs down to where they seem to stop, and extrapolate the fiducial to even fainter magnitudes in order to assess completeness. This

fiducial line is shown in the left-hand panel of Fig. 2 (in magenta, as in other figures of this article).

With KS2, we added artificial stars along this fiducial line with a flat distribution in m_{F606W} between magnitudes 24 and 32, and with a homogeneous spatial distribution across the field of view. Therefore, this fiducial line was defined on the observed WD CS of NGC 6752. We did not use theoretical models, nor we made assumptions about the WD CS location. We follow the prescriptions in Bedin et al. (2009, section 2.3), and correct our magnitudes (both real and artificial) for input-output systematic errors, which are negligible at $m_{\text{F606W}} \sim 24$, but become as large as ~ 0.2 mag at the faintest magnitudes. In the following, our magnitudes – for both artificial and real sources – are corrected for these effects.

Unfortunately, ASTs cannot track down all the possible sources of photometric systematic errors, the most important being related to CTE effects. Indeed, in the case of real sources, their photo-released charge is bitten out by the electron traps encountered in the detector that are filled during the read-out process, all the way down to the amplifier (up to ~ 2000 pixels). ASTs instead are just added values to the analysed images, and their artificial charge was never subjected to all the electron-traps which would lay along the read out process.

The recent ISR by Anderson et al. (in preparation) use an observing strategy to self-calibrate these CTE-related photometric errors, which requires multiple observations of the same field to be collected at ~ 90 degrees from each other. As our observations of NGC 6752 were not collected this way, a correction of this kind here is not possible. However, results in that work suggest that in the case of our data set, any possible systematic photometric error related to the CTE losses, could amount to ~ 0.1 mag, which therefore in the following we set as the limit of our photometric accuracy. We also note that F606W exposures have on average a 45 per cent higher sky-background level than those in F814W (~ 136 versus ~ 94 photo-electrons) making photometry F606W less vulnerable to CTE losses, as many more electron traps are filled. Furthermore, the number of images in F606W is more than twice the number available in F814W (precisely 112 versus 53), making m_{F606W} -magnitudes more robust measurements than those in F814W. For this reason, in the following, we will chose to analyse both the CMD and the LF in the m_{F606W} -magnitudes.

To establish whether an inserted star was recovered or not, we assumed that if an artificial star is not recovered within 0.753 ($\sim -2.5 \log 2$) magnitude (in both filters), and within 1 pixel from the inserted position (in both x and y detector coordinates), then the inserted artificial star was not recovered. The right-hand panel of Fig. 2 shows artificial sources as inserted (in magenta) and how they were recovered (dots).

The panels of Fig. 2, together, are used to define the region within which we will count the WDs of NGC 6752. Specifically, this region is defined by two lines (in green) drawn by hand, which are a compromise between including the observed WDs of NGC 6752 with large photometric scatter, and excluding the vast majority of field objects. We allowed for photometric errors very large around the magenta line, so that the bulk of the observed WDs (which might not follow exactly the magenta line), will still be included.

5 COLOUR-MAGNITUDE DIAGRAM AND SELECTIONS

Following the approach of Paper III, in Fig. 3, we show the impact of our progressive selection criteria on artificial stars, and then apply the criteria identically to real sources. Each panel is labelled in the top-right corner with an (a) for ASTs, or with an (r) for real

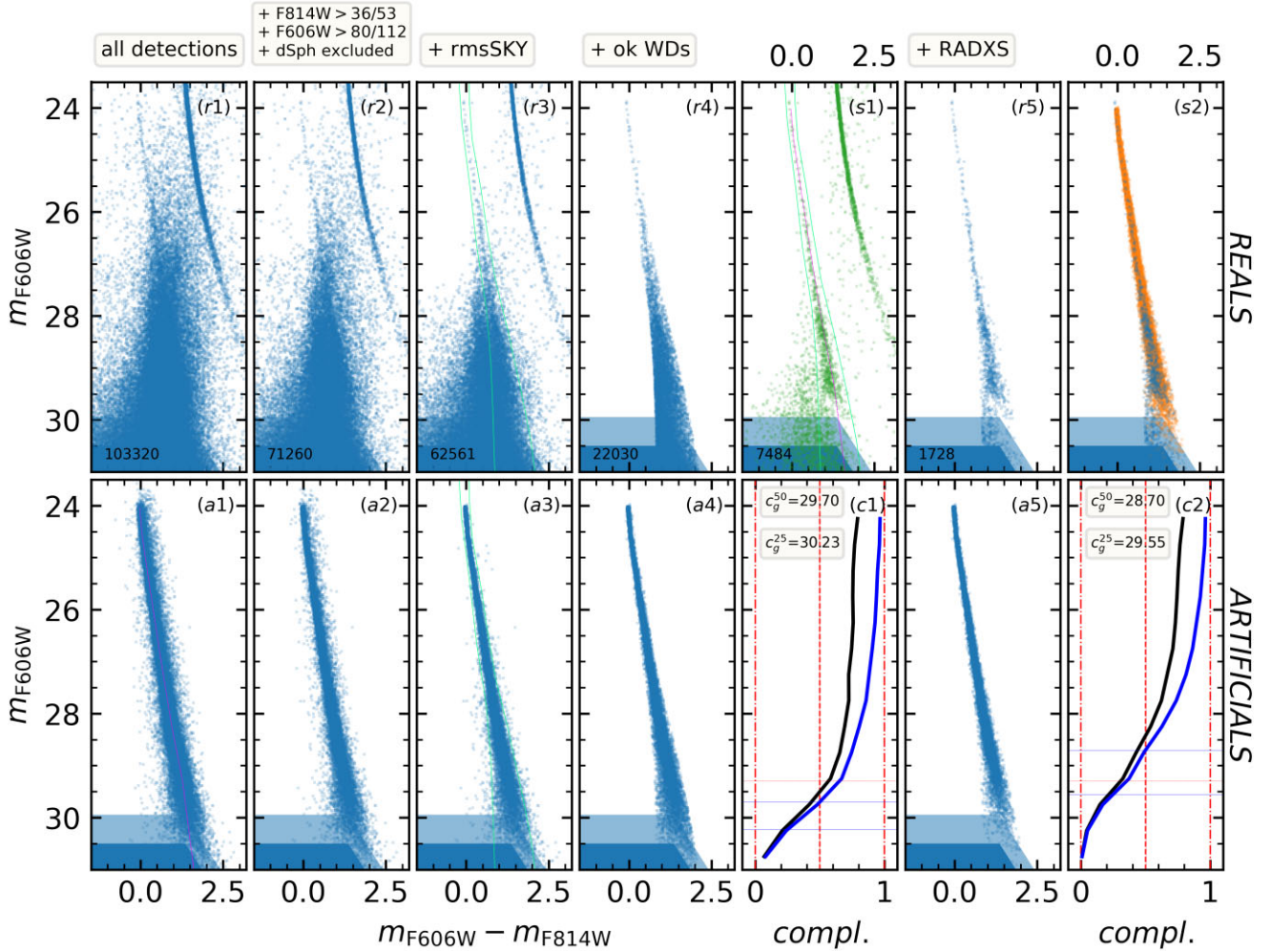


Figure 3. Panels show the progression of the cumulative selections adopted to obtain a sample of well-measured WDs along NGC 6752’s WD CS. From left-hand to right-hand panels from (1) to (5) for real stars [top (r_i) panels] and artificial stars [bottom (a_i) panels]. A thin line in magenta in panel (a_1) show where artificial stars were added. Panels (c) show the obtained completeness, not including (c_1) or including (c_2) the selection on RADXS, which is by far the most effective parameter to select well-measured point sources. For clarity in panel (s_1), we ‘show’ the effect of RADXS selection on stars also outside the WD region defined by the two green thin lines (see text). Finally, in panel (s_2), we show the direct comparison of real versus artificial stars, clearly not showing a sharp drop in their number below $m_{F606W} \simeq 29.4$.

sources. The goal of these selections is to find the best compromise between keeping the largest sample of well-measured WD members of NGC 6752 and rejecting the detections that are most likely spurious, or poorly measured stars, or field objects in the foreground and background of the cluster – be they either stars or galaxies.

The KS2 code sets a detection threshold for potential sources to be any positive local maximum in an image that is above 1σ of the local sky noise. Naturally, this choice results in the inclusion of a large number of non-significant detections, but it has the value of showing where the floor-level of the white noise is located. As in Paper III, we measured the background noise (1σ) in the two filters for regions with average low background and transformed these values into magnitudes, associating to these peaks the value of the PSFs’ central-pixel value (normalized to unity), and zero-pointing to the Vega-mag system. We show in panels (a) and (r) with shading the corresponding 5σ (light blue) and 3σ (dark blue) regions. In the following, we will not consider as significantly detected any source below the 5σ limits. Therefore, sources above these shaded

regions could still be poorly measured objects or even artifacts, but nevertheless they are solid detections.

Panel (a_1) show all the artificial sources as inserted (in magenta) and as they were recovered (blue dots).

In panel (a_2), we restricted our sources to those that fell in the part of the field that was observed in at least ~ 70 per cent of the F814W and F606W images. Given our large dither observing strategy, this leads to a significant reduction of the field of view that is used for this investigation.

Panel (a_3) further restricts us to a region of the field where the rmsSKY was consistent with the noise within empty patches of sky, i.e. to regions that were not dominated by the bright halo of luminous stars. This selection has only a marginal effect in rejecting sources in noisy regions of the field of view (FoV), but it has a great significance in establishing which regions are suitable (‘good’) for the detection of faint objects (thanks to the lower sky noise level). The computed completeness in ‘good’ regions is indicated by c_g , following Bedin et al. (2008).

In Panel (a4), we use the region enclosed within the two green lines in panel (a3) to reject all the ASTs recovered outside this region.

In the next panel, (c1), instead of a CMD, we show the magnitude versus completeness curve (i.e. the: number-of-recovered/number-of-inserted sources) with a black line, and the completeness limited to ‘good’ regions (c_g) as a blue line for the surviving artificial stars. This panel shows that inserted sources that passed these selections are 50 per cent-complete down to $m_{F606W} \simeq 29.7$, and 25 per cent-complete at $m_{F606W} \simeq 30.2$. However, we can recognize that while 50 per cent-completeness is well above the sky noise at 5σ , the 25 per cent-completeness is well below our 5σ minimum threshold for significant sources. In the following, completeness corrections are assumed to be reliable only for magnitudes brighter than the 5σ magnitude level expected for sky noise, even though we show some points below this.

In the next panel (a5), we show the result after selection with our most effective diagnostic to reject non-stellar objects, i.e. the RADXS parameter. This parameter is able to reject most of the PSF artifacts, diffraction spikes, extended sources, and field objects that have moved significantly more than the cluster member stars (about 1 mas) during the ~ 3.5 yr between first and last epochs, causing a blur of the shape of stars that do not move as cluster members.⁴

The bottom-right panel (c2) shows the completeness curves after the final selection on RADXS is also applied. Note that 25 per cent-level of the c_g -completeness remains well above the 5σ sky-level down to magnitudes $m_{F606W} \simeq 29.55$, i.e. below the location of the WD CS LF’s peaks as observed in Paper III.

In the top panels of Fig. 3, we have applied the very same selections defined for ASTs to the observed real sources. In panel (r5), we show the final sample for NGC 6752’s WD candidates. We also show the selection of two interesting real-star CMDs [labelled with (s)].

Panel (s1) shows the CMD for stars that passed all the selection criteria, but includes sources falling outside the region between the two thin green lines defined in panel (a3). This CMD includes non-WD candidates. Sources in this CMD are coloured in green, and will be used in Section 6 to derive a model of the field contamination within the WD region in the CMD. Note also that the fiducial line defined in Section 4 (and shown in magenta) well represents the mean observed CMD location for WD CS of NGC 6752.

Panel (s2) shows a direct comparison of the CMDs for sources surviving all the selection criteria: ASTs [as selected in panel (a5)] are shown in orange, while real sources [as selected in panel (r5)] are shown in blue. From this CMD, it is evident that the observed real WD CS does not extend to magnitudes as faint as those of the recovered ASTs – a clear indication that we have reached and passed the peak of the WD CS LF of NGC 6752.

6 PROPER-MOTION DECONTAMINATION

We now check whether the PMs derived for the faintest stars can be of any help to discriminate between field objects falling within the WD region of the NGC 6752’s CMD, and true cluster members. To this end, we combined the bulk of the first half of the data collected in ~ 2018.7 and obtain averaged positions for the sources, which are then compared with their averaged positions as measured in the second half of the data collected in ~ 2021.7 . In the following analysis, we will consider only sources shown in green in panel (s1)

of Fig. 3 (i.e. those surviving all the selection cuts described above, therefore considering sources outside the region of the WDs in the CMD), but for which it was possible to estimate a position at each of the two considered epochs.

Unfortunately, the absolute motion of NGC 6752 (relative to the field) is not very large: $(\mu_{\alpha\cos\delta}; \mu_{\delta}) = (-3.155; -4.010) \pm (0.008; 0.009)$ mas yr⁻¹ (Libralato et al. 2022), resulting in a combined absolute motion of just 5.102 ± 0.012 mas yr⁻¹. With a time-baseline of just ~ 3 yr, we can expect a separation of about ~ 15 mas between the cluster members and the faintest extra-galactic unresolved field sources that sit in an absolute rest frame. Given the ACS/WFC pixel-size of 49.72 mas, this amounts to a displacement of about 0.3 pixel, which is an easily measurable quantity for the brightest stars, but it is smaller than the measurements errors for those extremely faint sources that become significant only when combining several dozen individual images of the same epoch.

We provide a quantitative illustration of this situation in Fig. 4. We colour-code in blue stars surviving the WD selection defined by the two thin green lines, and in orange all other stars. In the top panels, we show vector-point diagrams of source displacements over the ~ 3 yr, in units of ACS/WFC pixels (of 49.72 mas) for 2D-displacements ($\Delta X, \Delta Y$) between the two epochs (2018.7 versus 2021.7). In the first, third, and fourth bottom-panels, we show CMDs. The second bottom-panel from left shows the observed 1D-displacement (ΔR summing in quadrature the displacements in X and in the Y axes) as a function of the observed source m_{F606W} -magnitude. Among bright stars, it is clear that there is a tight distribution in ΔR for cluster members, which remain distributed well below 0.1 pixel, and a broad tail toward higher ΔR , peaking between 0.2 and 0.4 pixels: these are field objects. However, at around magnitude $m_{F606W} \simeq 28$, the random positional-measurement errors (which are summed in quadrature for the two epochs) explode for fainter stars. And at around magnitude $m_{F606W} \simeq 28.5$, it clearly becomes impossible to disentangle members (with positional random errors around 0.4 pixels) from field objects (with a relative average displacement of ~ 0.3 pixels).

With the observed bulk of stars at different magnitude intervals, we define the PM selections for objects consistent with the PM errors at the various magnitudes. This arbitrary selection is illustrated by the thick-grey line. Stars satisfying this selection (grey step-line) are indicated with circles, while stars not passing this selection or fainter than $m_{F606W} \simeq 30$ are indicated with crosses. The panels on the right of this panel show the vector-point diagram and the CMD for sources to the left or to the right of the gray-line criterion, keeping the blue colour-code for WD candidates. The PM selection, while useful to reject outliers and objects with large PMs, is completely useless at separating WDs and field objects below $m_{F606W} \simeq 28.5$, simply because the separation between field and members is much smaller than measurement errors below $m_{F606W} \simeq 28.5$. In the next section, we introduce a work-around to properly define the WD CS LF of NGC 6752 down to its faintest magnitudes.

7 THE CORRECTED WD CS LF

As demonstrated in various *HST* studies, including those that made use of the *Hubble Ultra Deep Field* (e.g. Bedin et al. 2008, 2009, 2010), the vast majority of contaminants in the CMD aligned with the cluster’s WD CS are blue galaxies that are relatively easy to reject with shape parameters such as RADXS. However, at fainter magnitudes, unresolved blue galaxies become increasingly indistinguishable from stars. Although these faint unresolved blue point-sources fall in a region just outside the WD CS (e.g. Bedin et al. 2009 and discussion, or fig. 3 of Paper III), a number of

⁴Since cluster members are used to compute the transformations, they do not move within errors, especially towards faint magnitudes, where random errors dominate uncertainties.

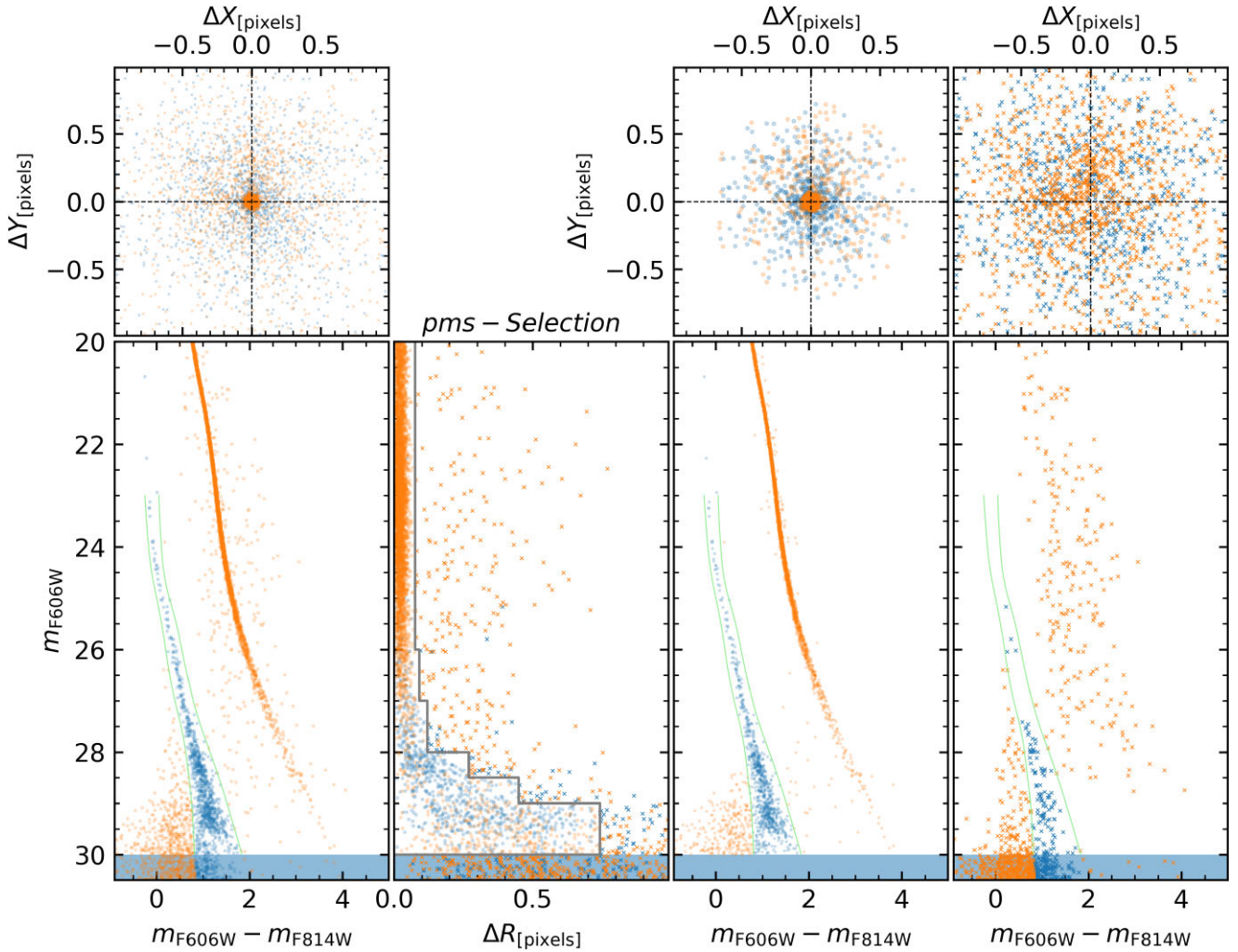


Figure 4. (*Bottom-left:*) CMD of the sources in panel (s1) of Fig. 3; here sources within the WD-region (between the two green lines) are colour-coded in blue and all the other sources in orange. Shading below $m_{F606W} = 30$ marks the limit of significant detection for the sources of interest. (*Bottom-mid-left:*) 1D displacements, ΔR , as function of m_{F606W} . Bright stars (down to $m_{F606W} \sim 28$) show a ΔR distribution with a tight dispersion (<0.1 pixels) plus a tail with a much broader dispersion peaked around 0.3 pixels. We arbitrarily define two regions, indicated by the grey line, one that enclose the bulk of the ΔR at different magnitude bins (dots), and the other with objects of larger ΔR (small crosses). The next two bottom-panels on the right show the CMDs for the stars within and beyond the grey line. Clearly neither of the two CMDs are made only of members or only of field objects (see text). (*Top:*) 2D vector point diagrams of the ~ 3 yr-displacements for the samples shown in the corresponding panels below.

blue stars in the Galactic field contaminate the LF. Unfortunately, this contamination appears far from negligible, and for a reliable study of the WD CS LF of NGC 6752 some correction of residual contaminants should be performed.

In this section, we develop a simple model to correct for field contamination. The process is illustrated in Fig. 5 and described in the following. In the first panel of this figure, from left to right, we show the CMD of sources defined in panel (s1) of Fig. 3, and use the two lines of Fig. 3 to define on the CMD what we will refer to as the ‘WD-region’. We then defined two other regions with the identical colour-width at each magnitude of the CMD WD-region, but one at bluer colours (to the left of the WD-region), and one at redder colours (to the right of the WD-region). In the following, we refer to these as ‘Blue-’ and ‘Red-regions’. Next, we counted the sources within each of these three regions, and produced in the next panel to the right of the LFs of each. We visualize the LF for those WD candidate objects observed within the WD-region, and the Blue- and Red-regions, with error bars that reflect statistical Poisson errors.

In the simplest model for the contaminant distribution on the CMD’s WD-region, we assume that: (i) there are no WDs of NGC 6752 within the Blue- or Red-region; and (ii) the number of contaminants within the CMD WD-region is the average of the number of objects observed in the Blue- and Red-regions, at the various magnitudes. We show this model in Fig. 5, with corresponding errors estimated by linear propagation of Poisson noise.

In the third panel of Fig. 5, we compare the observed LF to the resulting WD LF corrected for the field-contamination model. The field-corrected LF is simply the observed LF minus the field model, with errors propagated linearly. Interestingly, in the faintest bins, $m_{F606W} = 29.6$ and 30, the field correction brings the observed WD CS LF to zero within the uncertainties. As this is what one would expect when the LF drops out – in spite of the naïve assumption for this distribution of contaminants – this is a rather reassuring feature of the goodness and validity of the model. Finally, in the last panel to the right, we show the completeness-corrected and field-corrected WD

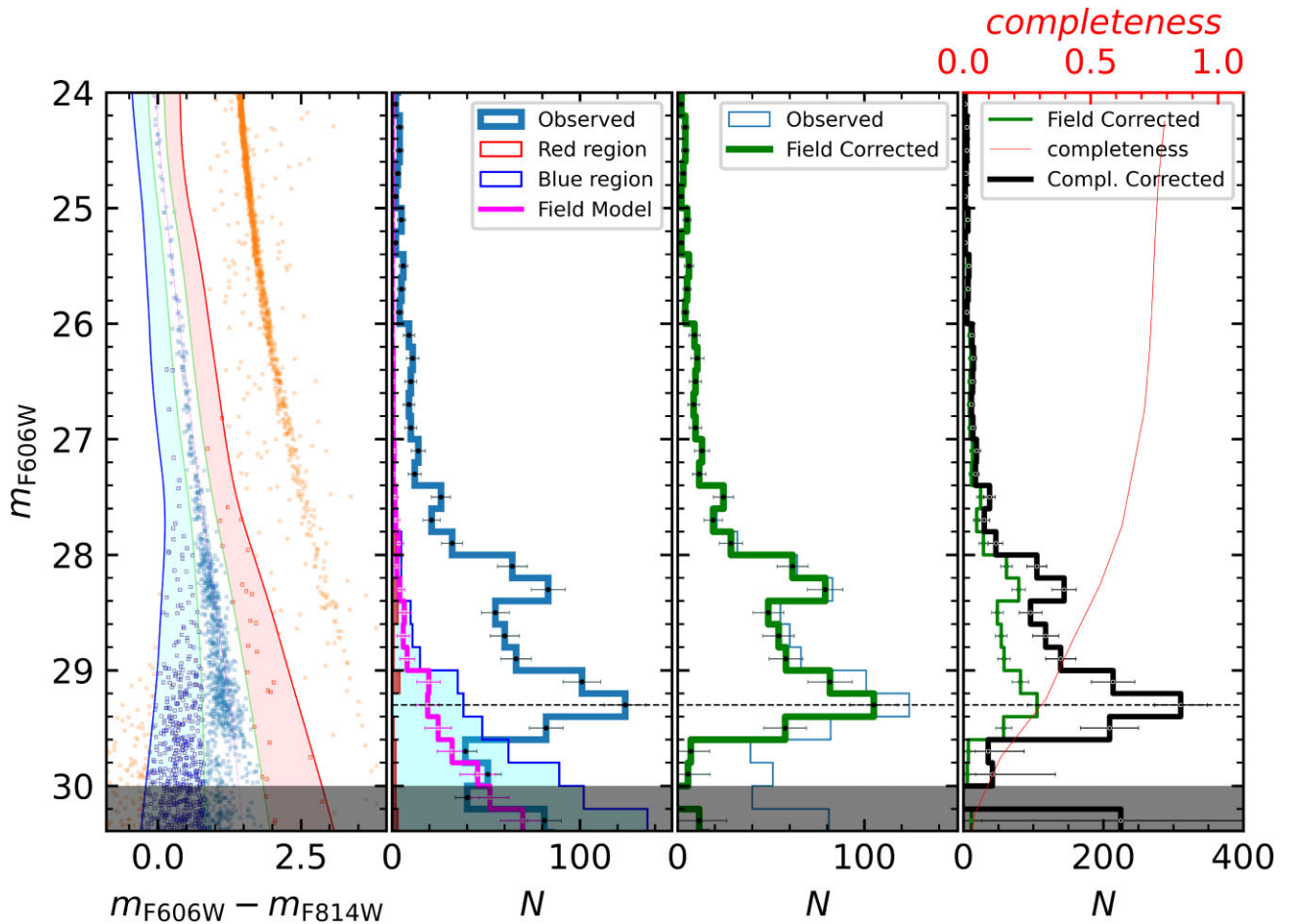


Figure 5. (Left:) The CMD where we define three regions along the WD CS. The shaded region in azure indicates sources passing the selections on the blue side of the WDs, the region shaded in red, the ones redder. Next panels are all WD CS LFs, where the histograms show the number of sources per magnitude interval, for observed stars within the WD region and for stars in the two shaded regions (see the legend). Our model for field distribution is the histogram in magenta. This is a simple average of the blue and red LF. This simple model is then subtracted from the observed LF, and shown in the third panel from left to right. Note how the faintest bins of the corrected WD CS LF are consistent with zero within the noise. Finally, the observed field-corrected WD CS's LF was corrected for completeness, and shown in black. Grey-shaded regions indicate levels where findings and completeness become unreliable (see text). Errors where linearly propagated and then corrected for completeness.

CS LF for NGC 6752. The errors on this LF were also corrected for completeness with a simple approximation, a linear propagation of the errors. As supplementary online material, we also release all data (completeness, observed LF, errors, CMD, and photometric errors) to enable other groups to independently perform their own analyses. The WDLF in the right-hand panel of Fig. 5 is the one used in the next section for the theoretical analyses.

8 MODELLING THE OBSERVED WD CS AND LF

As in Paper III, a preliminary step to theoretically model the observed CS is to have estimates of the cluster parameters, such as distance and reddening. We employed here the same distance modulus ($m - M_0 = 13.10$ and reddening $E(B - V) = 0.05$, as in Paper III; this distance is consistent with the recent determination ($m - M_0 = 13.08 \pm 0.02$ by Baumgardt & Vasiliev (2021), while the reddening is consistent with $E(B - V) = 0.046 \pm 0.005$ determined by Gratton et al. (2005). We compared theoretical isochrones from the lower MS to the tip of the red giant branch (RGB), and a zero-age HB (ZAHB) sequence, to the optical CMD of the cluster central

regions from Nardiello et al. (2018, N18), merged with the deep photometry for the external field studied in this work, as shown in Fig. 6. These two photometries agree well, and the richer central field populates the post MS phases and the bright part of the WD CS. We employed α -enhanced ($[\alpha/\text{Fe}] = 0.4$) isochrones and ZAHB models by Pietrinferni et al. (2021) for $[\text{Fe}/\text{H}] = -1.55$ – consistent with $[\text{Fe}/\text{H}] = -1.55 \pm 0.01 \pm 0.06$ (random + systematic) determined by Gratton et al. (2005) – and an initial helium mass fraction $Y = 0.248$. In this comparison with the observed CMD (and also for the modelling of the WD sequence), we applied extinction corrections to the $F606W$ and $F814W$ filters that depend on the model effective temperature, calculated as in Bedin et al. (2005).

Fig. 6 shows the overall good agreement between the theoretical models and the observed sequences on the MS, RGB, and ZAHB. The plume of stars that departs from the ZAHB towards higher luminosities is made of objects coming from the blue tail of the HB that are evolving towards the asymptotic giant branch (AGB). This comparison provides us with a MS TO age determination; isochrones with ages equal to 12.5 and 14.0 Gyr match the TO region of the CMD, and also bracket nicely the luminosity range of the subgiant branch that is very sensitive to age. We also display a 12.5 Gyr

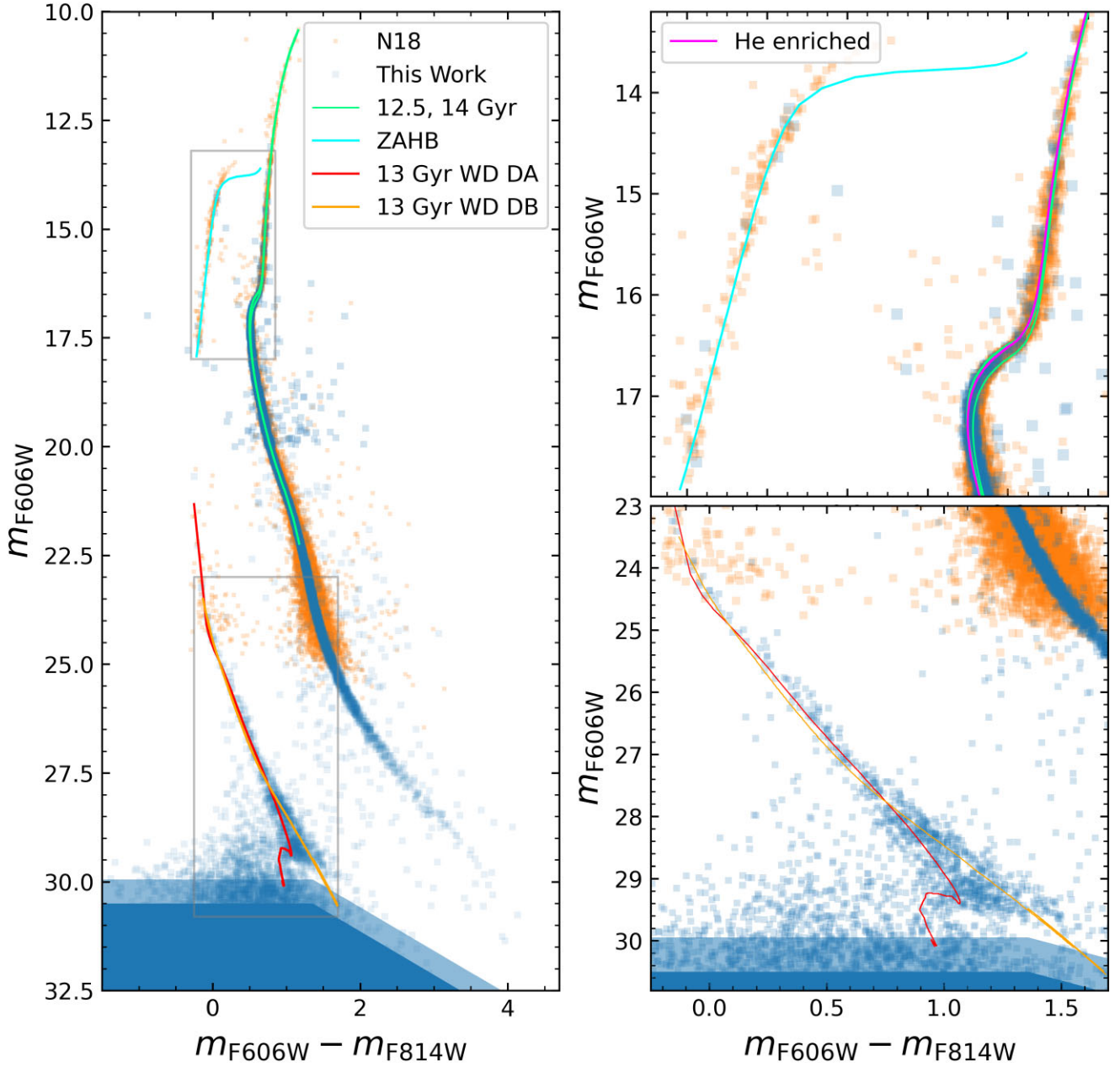


Figure 6. (Left): Cluster CMD compared to theoretical isochrones. In orange, we display the catalogue from the central region of NGC 6752 by Nardiello et al. (2018, N18); in blue, the deep photometry for the external field studied in this work. The two sets of data agree; the richer central field of N18 is based on much shallower observations of a much more crowded region, so they become incomplete at $m_{F606W} \sim 25$, but show well populated the bright and fast evolutionary phases, in particular: the RGB, the HB, and the bright part of the WD CS. The displayed ZAHB and theoretical isochrones are labelled. The 13 Gyr hydrogen envelope WD isochrone is denoted as DA, and the helium envelope counterpart is denoted as DB. (Right): zoom-in of the gray-rectangular regions indicated in the CMD of the left-hand panel. The Top panel shows the MS + HB + RGB region with the He-enriched isochrone (in magenta). The Bottom panel focuses on the faint WD CS.

old isochrone with initial $Y = 0.275$ (keeping the metal content fixed) to show the effect of the helium spread due to the presence of multiple populations in the cluster. Just as a reminder, the presence of multiple populations in a GC is revealed by (anti)correlated variations of the abundances of C, N, O, and Na (and sometimes Mg and Al), plus variations of the initial helium mass fraction Y (see e.g. Renzini et al. 2015). A typical GC-like NGC 6752 hosts a population of stars with ‘normal’ initial helium abundances and α -enhanced metal abundance patterns similar to those of field halo stars at the same $[\text{Fe}/\text{H}]$ – we denote it as first population – together with a

population of helium-enriched stars displaying a range of N, Na, and He overabundance, coupled to C and O depletion compared to field stars at the same $[\text{Fe}/\text{H}]$ – we denote this as second population. When observing in optical filters and as long as the sum of the CNO abundances is the same in the first and second population – as generally from spectroscopy – stellar models calculated with the standard helium and α -enhanced composition of the first population are appropriate to study observations of clusters’ second populations (see the review by Cassisi & Salaris 2020, and references therein).

In NGC 6752, the average difference ΔY among the cluster multiple populations is estimated to be small, ~ 0.02 – 0.03 (Cassisi et al. 2014; Nardiello et al. 2015; Milone et al. 2018), and the $Y = 0.275$ isochrone well represents the cluster second population. We can clearly see that the effect of the enhanced He on the TO region of the isochrones is minor, and does not appreciably affect the TO age estimates. Also, as discussed by Cassisi et al. (2014), the helium-rich second population is located towards the fainter blue end of the HB, a region where the corresponding helium-rich ZAHB would overlap with the $Y = 0.248$ ZAHB shown in Fig. 6.

Moving now to the WD CS, Fig. 6 also shows a comparison between the observed CMD and two 13 Gyr WD isochrones, employing the same distance modulus and reddening as for the comparison to the MS, RGB, and ZAHB CMD. The isochrones have been calculated from the carbon-oxygen core WD cooling tracks by Salaris et al. (2022); specifically, the models with hydrogen or helium envelopes and metal-poor progenitors (see Salaris et al. 2022, for details) computed with the Cassisi et al. (2007) electron conduction opacities (see later for more details on this). We employed the same initial-final mass relation (IFMR) by Cummings et al. (2018) used to calculate the carbon/oxygen profiles of the WD models, and the progenitor lifetimes from Salaris et al. (2022) evolutionary tracks with the appropriate metallicity ($[\text{Fe}/\text{H}] = -1.55$). The helium envelope isochrone does not reach the faint end at this age, because of limitations of the input physics of the models. Due to these limitations, the calculations of WD models above $\sim 0.6 M_{\odot}$ could not reach cooling ages comparable with the ages of GCs. However, the fainter point along the helium envelope isochrone is still much fainter than the magnitude of the cut off of the observed LF, because of the faster cooling times compared to hydrogen envelope models.

Fig. 6 displays an overall good agreement between observations and isochrones from the bright to the faint end of the CS. Below $m_{\text{F606W}} \sim 28$, the hydrogen envelope isochrone becomes increasingly offset towards bluer colours compared to observations, by up to ~ 0.1 . Given that hydrogen envelope WDs are present along the cluster CS (see e.g. Renzini et al. 1996; Moehler et al. 2004; Chen et al. 2022) we believe that systematic offsets of this order are compatible with the residual uncertainties in the photometry of the faintest sources, which likely affect mostly the F814W filter (see end of Section 4).

Interestingly, the observed breadth of the faintest part of the WD CS is larger than that inferred from ASTs, but bracketed by the two shown isochrones. There also seems to be some structure with a possible hint of a gap around $m_{\text{F606W}} \sim 28.8$; however, only data with better signal to noise would be able to investigate the exact shape of these putative features.

Our analysis of the WD LF focuses on isochrones from models with hydrogen envelopes, as current evidence indicates that at least the bright WDs in NGC 6752 and other GCs are generally found with hydrogen atmospheres (Renzini et al. 1996; Moehler et al. 2004; Davis et al. 2009; Chen et al. 2022). This assumption is reinforced by the fact that, as we describe below, the shape of the empirical LF agrees well with a population of WDs with hydrogen envelopes. We will briefly investigate the impact of the possible presence of WDs with helium envelopes in Section 8.4

To study in more detail the consistency of the ages from the CS and the MS TO, we need to model the WD LF. Before doing so, we need to consider the effect of the HB morphology of this cluster on the WD population.

As briefly discussed in the introduction, the HB of NGC 6752 shows a pronounced extension to the blue, with objects increasingly less massive when moving towards bluer colours, because of thinner hydrogen-rich envelopes around the helium core. Chen et al. (2022)

have discussed in detail how the bluer HB stars have envelope masses too small to reach the AGB after the end of core helium burning, hence they do not experience the thermal pulses and third dredge-up. These objects reach the WD stage with a residual hydrogen envelope thick enough to sustain stable thermonuclear burning (Althaus et al. 2015). For redder HB stars with masses above $\sim 0.56 M_{\odot}$, the post-HB evolution leads instead to the AGB phase and the thermal pulses. The resulting WDs have thinner outer hydrogen layers of ‘canonical’ mass thickness equal to $\approx 10^{-4} M_{\text{WD}}$, where M_{WD} is the total WD mass, not able to support efficient hydrogen burning (Althaus et al. 2015; Chen et al. 2021). As a consequence, the bright part of the cluster CS is populated by two different populations of WDs; a ‘slower’ population supported by envelope hydrogen burning, and a ‘canonical’ population which is cooling faster, without any substantial contribution from nuclear burning. The number ratio of these two populations is determined by the mass distribution of the HB stars which, in turn, is controlled by the distribution of initial helium abundances among the cluster’s multiple populations, as studied by Cassisi et al. (2014). Down to $m_{\text{F606W}} \sim 25.1$, the magnitude range studied by Chen et al. (2022), this number ratio ‘slow’/‘canonical’ is equal to $\sim 70/30$, a value that is expected to progressively decrease at fainter magnitudes because these WDs formed earlier in the life of the cluster, when stars along the HB were more massive and increasingly less blue. At $m_{\text{F606W}} \sim 27.3$, corresponding to cooling times of 3–3.5 Gyr, we expect to have only ‘canonical’ WDs because their HB progenitors all had masses above $0.56 M_{\odot}$ (see the discussion in Chen et al. 2022).

In our deep photometry, which covers the outer regions of the cluster, the bright part of the LF is not well populated because of the sparser density (necessary to study faint objects). Therefore, we will need to perform a comparison with model predictions considering $m_{\text{F606W}} \geq 27.3$, where the number of objects is at least 20 per bin. Coincidentally, this is also the magnitude range where only canonical DA WDs are expected to populate the CS.

We have calculated theoretical WD LFs starting from isochrones computed as described before, using as reference a Salpeter-like power law for the WD progenitor mass function with exponent x set to -2.3 . The WD models we employ have negligible efficiency of Hydrogen burning in their envelopes (the mass thickness of the hydrogen envelope is equal to $10^{-4} M_{\text{WD}}$). To properly include the photometric errors, we followed a Monte Carlo approach; we have first randomly drawn progenitor masses according to the chosen mass function and determined the magnitude and colour of the corresponding synthetic WD by interpolation along the isochrone, after applying the distance modulus and extinction. The magnitude and colour have then been perturbed by Gaussian random errors with 1σ values determined from the mean 1σ errors at the magnitude of the synthetic object, as obtained from the photometric analysis. We finally calculated the LF from the magnitude distribution of the resulting sample of synthetic WDs. For each LF calculation, we have drawn 100 000 synthetic stars to minimize the statistical error in the star counts and normalized the theoretical LFs matching the total number of objects in the empirical LF in the bins centred between $m_{\text{F606W}} = 27.3$ and 27.9 (130 objects).

Fig. 7 shows theoretical LFs for the three ages that match the position of the peak and subsequent drop of the star counts at the faint end of the empirical LF. As well-known, the magnitude of this feature is a diagnostic for the total age (cooling age plus progenitor age) of a cluster’s WD population. The fainter the magnitude, the older the WD population. The range of ages inferred from the theoretical LFs is between 12.7 and 13.5 Gyr, completely consistent with the cluster age derived from the MS TO and subgiant branch, as shown in Fig. 6.

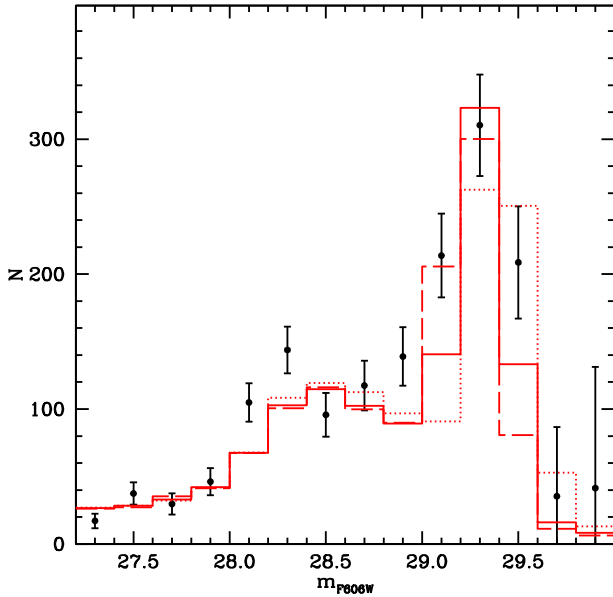


Figure 7. Observed WD LF compared to theoretical counterparts for ages equal to 12.7 Gyr (dashed line), 13.0 Gyr (solid line), and 13.5 Gyr (dotted line), respectively. The theoretical LFs have been calculated using for the progenitors’ mass function a power law with exponent $x = -2.3$, and are normalized by matching the total number of stars of the empirical LF between $m_{F606W} = 27.3$ and 27.9 (see text for details).

The shape of the theoretical LFs follows closely the shape of the empirical counterpart. In the magnitude range used for the normalization, the theoretical LFs display a very mild increase of the number counts with increasing magnitude, as observed. There is a sharper increase of the number counts towards fainter magnitudes, with a roughly flat portion down to the bin centred at $m_{F606W} = 28.9$.

This magnitude range of the theoretical LF is populated by objects with mass – and progenitor mass – increasing only very slowly with magnitude, the average WD mass being around $0.56M_{\odot}$, as shown in Fig. 8. The number of stars per magnitude bin will therefore be mainly determined by the local cooling speed, with a smaller contribution from the choice of the progenitor mass function. Starting from $m_{F606W} \sim 28.0$, the cooling speed slows down because of the onset of crystallization, hence the local increase of the number counts.

Beyond $m_{F606W} = 29.0$, both theoretical and observed LFs display a steep increase of the number counts that peak at $m_{F606W} = 29.3$, followed by a sharp drop at fainter magnitudes. In this magnitude range, the LF is populated by all other more massive WDs, originated from progenitors with initial mass above $\sim 1.5M_{\odot}$, as shown in Fig. 8. These objects have reached fainter magnitudes because of their longer cooling times (shorter progenitor lifetimes). Their piling up in a relatively narrow magnitude range explains the appearance of the peak and cut-off in the LF at the bottom of the CS. As a consequence of this pile-up of WDs originated by progenitors with a large range of initial masses, a variation of the exponent of their mass function can have a major impact on the predicted WD number counts in this magnitude range.

As an aside, we note that Fig. 8 shows a non-monotonic relation between WD (and progenitor) mass and magnitude at the faint end of the theoretical LF. This is a consequence of the fact that at these luminosities the model cooling times do not increase monotonically with increasing WD mass. This is due to the interplay of the onset (in terms of luminosity) of crystallization that depends on the WD mass (higher masses start crystallizing earlier), and the associated time

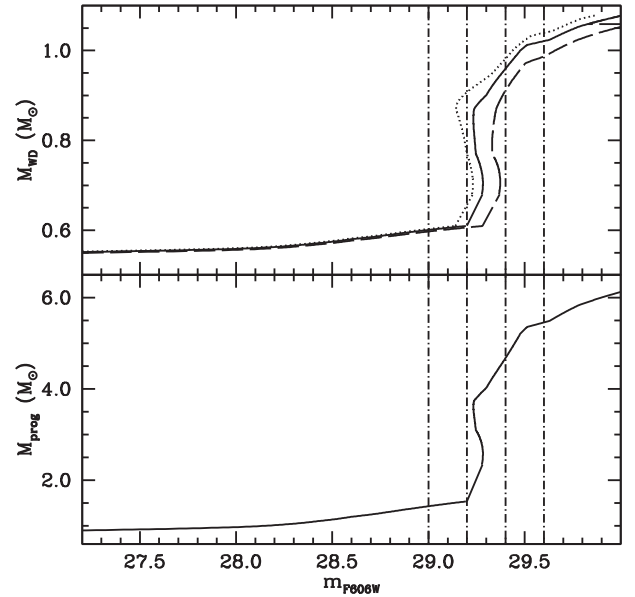


Figure 8. Distribution of the WD masses (in solar mass units – top panel) as a function of the m_{F606W} for the LFs displayed in Fig. 7 (keeping the same line-type code for the different ages). The bottom panel displays the corresponding distribution of the initial progenitor masses (in solar mass units) along the 13 Gyr LF. The dash-dotted vertical lines in the two panels mark the boundaries of the three magnitude bins that enclose the region of the peak and subsequent cut-off of the empirical LF.

delays, which are in turn dependent on the WD mass (because of the different CO profiles) and luminosity (energy released at higher luminosities induces shorter time delays).

It is very important to notice also that the shape of the WD LF NGC 6752 is almost identical to the shape of the LF of the WD CS of M 4 (see fig. 10 of Bedin et al. 2009), despite the different metallicity and HB morphology (redder) of this more metal-rich cluster. In M 4, the WD LF also displays a very mild increase of the number counts towards fainter magnitudes, followed by a sharper increase and a roughly flat portion, before the pile up at the bottom end of the CS. These general trends can also be seen in the WD LFs of the metal-poor cluster NGC 6397 (with a blue HB morphology) and the red HB, metal-rich cluster 47 Tuc (see fig. 2 of Hansen et al. 2013) even though in this latter cluster the magnitude range spanned by the roughly flat portion of the LF is narrower, likely because of its younger age.

One important aspect to consider in the interpretation of the WD LF is the role of the progenitor mass function and the possible complexities associated with the effects of the cluster dynamical evolution on this mass function. In particular, as a result of the interplay between internal dynamical processes and the external Galactic tidal field, GCs gradually lose stars that escape beyond the cluster’s tidal radius. This mass-loss preferentially affects low-mass stars and may significantly alter the cluster’s global mass function (see e.g. Vesperini & Heggie 1997; Baumgardt & Makino 2003). This process may therefore plays a role in determining the present-day mass distribution of WDs by altering the mass function of their progenitors before the WD formation and, more directly, by causing the preferential loss of low-mass WDs after their formation. In addition to these effects, it is important to consider the gradual segregation of massive stars towards the cluster’s central regions and the outward migration of low-mass stars driven by the effects of two-body relaxation. Since observations do not typically cover

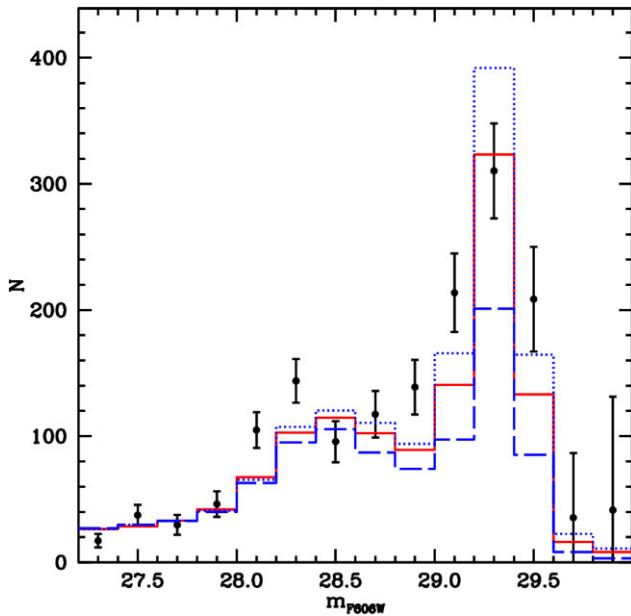


Figure 9. As Fig. 7, but showing three theoretical LFs for an age of 13 Gyr, calculated with exponents of the progenitor mass function equal to $x = -2.1$ (dotted line), -2.3 (solid line, the same LF of Fig. 7), and -2.8 (dashed line), respectively.

the entire radial extension of a cluster, a detailed model of the WD mass distribution would also require proper consideration of the effects of segregation and local variations of their mass distribution. Modelling the combined effects of these processes is non-trivial and would require an extensive suite of simulations specifically aimed at reconstructing the dynamical history of the cluster, and at exploring the implications for the mass distribution of its WD population. This is beyond the scope of this paper, but we emphasize that part of the discrepancies between the observed and the theoretical WD LFs shown below might be ascribed to the dynamical effects we have discussed.

8.1 The impacts of the cluster’s stellar mass function and initial-final mass relation

In order to provide a general illustration of the dependence of the WD LF on the stellar MF, in Fig. 9, we show the effect of changing the exponent x of the progenitor’s mass function in the calculation of the 13 Gyr LF. The magnitudes of peak and cut-off are not affected, but the number counts are. An exponent $x = -2.15$ matches better the observations compared to the $x = -2.3$ case. The total number of stars in the three bins centred at $m_{F606W} = 29.1$, 29.3 (the magnitude of the peak of the observed LF), and 29.5 of the empirical LF is 733 ± 64 , where the 1σ error has been calculated by propagating the 1σ errors on the number counts of the individual bins; the 13 Gyr theoretical LF calculated with $x = -2.1$ has 723 objects in this same magnitude range – consistent with the observations within less than 1σ – whilst only 600 objects populate the LF calculated with $x = -2.3$. In the range between $m_{F606W} = 28.1$ and 28.9 , the empirical LF has 601 ± 38 objects, whereas the theoretical counterpart calculated with $x = -2.1$ has 497 objects (477 objects in the LF calculated with $x = -2.3$), an underestimate of more than 2.5σ . This latter discrepancy might hint at some significant underestimate of the cooling time-scales of models in this magnitude range; in fact, a qualitatively similar discrepancy along the flat part of the LF before the pile up at the

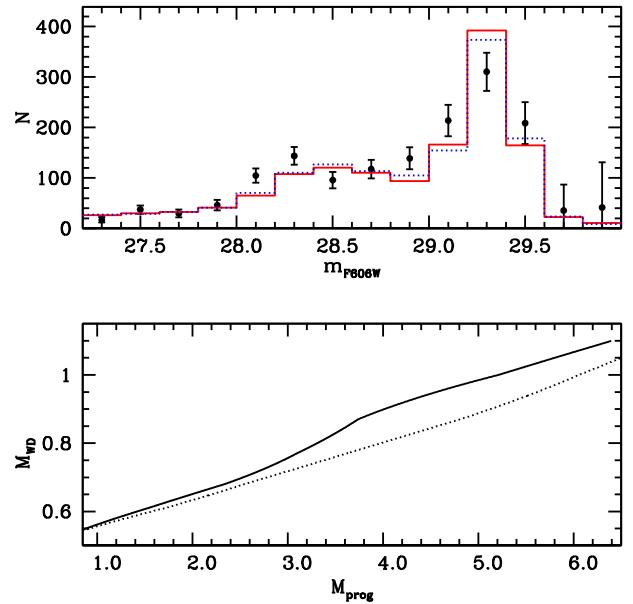


Figure 10. The top panel is as Fig. 9 but showing the 13 Gyr theoretical LF calculated with $x = -2.1$ (solid line) plus a coeval one calculated using the Salaris et al. (2009) and $x = -1.95$ (dotted line). The bottom panel compares the IFMRs used in the calculation of the LFs in the top panel.

end of the CS can be seen in the comparison of theoretical LFs with observations in M 4 (see fig. 13 of Bedin et al. 2009).

In addition to the stellar-mass function and its evolution driven by cluster dynamics, the IFMR may also play a key role in determining the WD LF. In Fig. 10, we display the results of another numerical experiment. It shows the 13 Gyr LF of Fig. 9 calculated with $x = -2.1$, plus a coeval LF computed by employing this time the linear IFMR by Salaris et al. (2009) and a progenitor mass function with exponent $x = -1.95$. The two theoretical LFs are almost identical, despite the different values of x and choice of IFMR. The number of objects around the peak of the observed LF is 707 for the LF calculated with the IFMR by Salaris et al. (2009), whilst in the range between $m_{F606W} = 28.1$ and 28.9 , this LF contains 527 objects, a number that is within 2σ of the observations.

To summarize, the quality of the agreement between observed and theoretical star counts along the LF depends somehow on the choice of both the progenitor mass function and of the IFMR, but the magnitudes of the LF peak and cutoff are much more solid prediction of theory. This confirms that the consistency between MS TO and CS ages is robust.

8.2 The impact of the electron conduction opacities

The empirical WD LF allows us to also test WD models calculated with the recent electron conduction opacities by Blouin et al. (2020). These authors have published improved calculations for H and He compositions in the regime of moderate degeneracy, which they have combined with the Cassisi et al. (2007) calculations to include the regime of strong degeneracy and cover the full parameter space necessary to stellar modelling. The opacities at the transition between moderate and strong degeneracy are still uncertain (see e.g. Blouin et al. 2020; Cassisi et al. 2021), but they are crucial for modelling WD envelopes and predicting the correct cooling times. As investigated by Cassisi et al. (2021), different ways to model this transition region

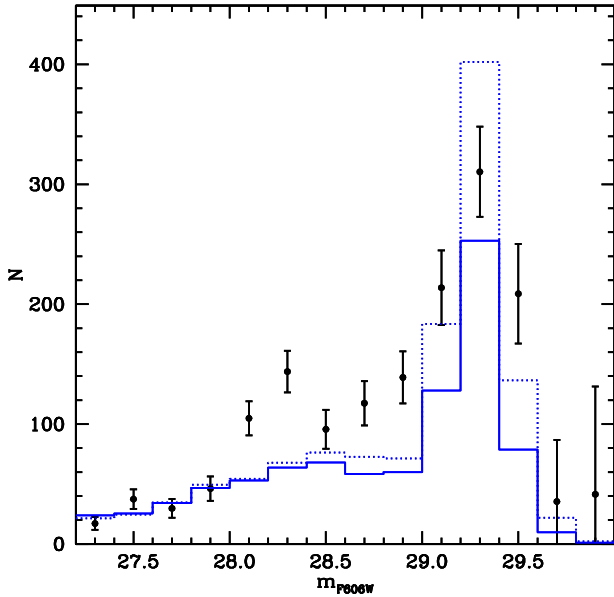


Figure 11. As Fig. 7, but showing two 10.7 Gyr theoretical LFs calculated using the Blouin et al. (2020) electron conduction opacities, and exponents of the progenitors mass function equal to -2.3 (solid line) and -1.8 (dotted line), respectively.

give a spectrum of values of the opacity that vary by up to a factor of ~ 2.5 in the regime relevant to WD envelopes.

We have calculated WD isochrones and LFs as discussed before, this time employing the set of Salaris et al. (2022) WD models computed with the Blouin et al. (2020) conductive opacities. Fig. 11 compares the observed LF with two of these new theoretical LFs for an age of 10.7 Gyr, computed with two different exponents of the progenitor mass function ($x = -1.8$ and $x = -2.3$, respectively). The age required to match the observed magnitude of the peak of the LF is now in disagreement with the age from the MS TO. Also the number counts in the magnitude range between $m_{F606W} \sim 28$ and ~ 29 , are largely underestimated, even for the LF calculated with $x = -1.8$, which compares well with the total number of objects around the peak of the empirical LF (721 objects in the theoretical LF). This is all consistent with the fact that, apart from the early stages of cooling, models calculated with Blouin et al. (2020) opacities evolve much faster than models computed with Cassisi et al. (2007) opacities (Salaris et al. 2022).

8.3 The impact of the cluster’s multiple populations

We have discussed already how the presence of multiple populations in the cluster impacts the bright part of the CS, because of the slowly cooling WDs supported by hydrogen burning in their envelopes, which are produced by the bluer HB progenitors (see Chen et al. 2022). In the magnitude range we are studying here, these are no longer present, but the existence of multiple populations can still potentially have an impact.

In this cluster, we have a ~ 70 per cent fraction of stars presently evolving in pre-WD phases with an initial value of Y larger by typically $\Delta Y = 0.02$ – 0.03 compared to the remaining 30 per cent of stars (Milone et al. 2017, 2018). When we assessed the impact of this second population on the MS TO ages, we calculated WD isochrones and LFs using progenitor lifetimes from Pietrinferni et al. (2021) models with initial $Y = 0.275$. In the assumption that the IFMR is unaffected by this small variation of the initial He of the progenitors,

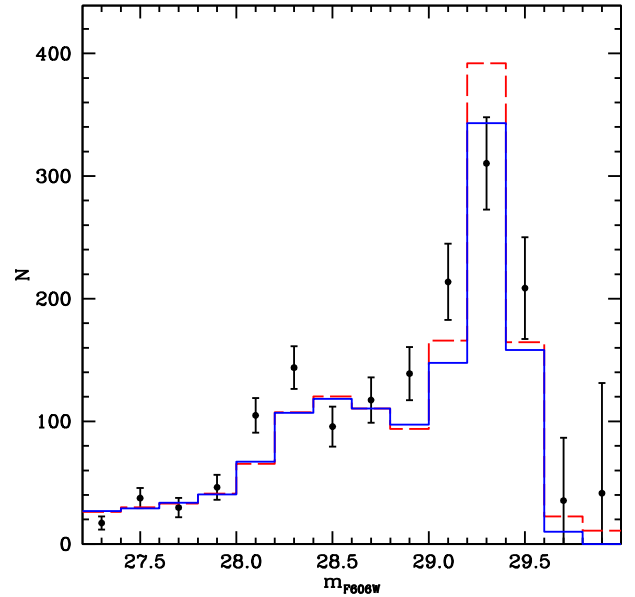


Figure 12. As Fig. 7, but showing two theoretical LFs for an age of 13 Gyr. The dashed line displays a LF calculated with helium-normal progenitors and mass function exponent equal to -2.1 (the same LF displayed as a solid line in Fig. 10), whilst the solid line shows a calculation with the same mass function but an initial $Y = 0.275$ for the progenitors.

we have verified with test calculations that the CO stratification of the WD models is also insensitive to such a small ΔY . Fig. 12 shows how such a small helium enhancement in the progenitors has a minor impact on the resulting WD LF. For a fixed age and exponent of the progenitor mass function, after normalization, the second population WD LF is extremely similar to the LF of first population WDs. Differences are minimal around the peak of the LF, hence comparisons of the empirical LF with that of a composite population made of first and second population WDs will give results essentially equivalent to the case of using models for just first population WDs.

8.4 The impact of helium envelope WDs

So far, we have studied the WD LF using only models with hydrogen envelopes. The top panel of Fig. 13 compares with observations the 13 Gyr LF populated by hydrogen envelope models and progenitor mass function exponent $x = -2.1$ of Fig. 10, and an LF populated by helium envelope models. This latter LF had been computed as described before, using Salaris et al. (2022) models with pure helium envelopes.⁵

The shape of the LF populated by helium envelope models is very different from the observed one. It stays essentially flat from the magnitude range chosen for the normalization down to $m_{F606W} \sim 29.5$, completely at odds with observations. Due to the generally faster cooling times of helium envelope models, in this magnitude range, this LF is populated by objects with practically constant mass, around $0.55M_{\odot}$, with progenitor initial masses in a relatively narrow mass range (the more massive objects appear at much fainter magnitudes).

⁵We notice that Bergeron et al. (2019) have shown how for WDs in the *Gaia* catalogue with helium-dominated atmospheres, the presence of a small percentage of hydrogen is required to determine more accurate stellar parameters.

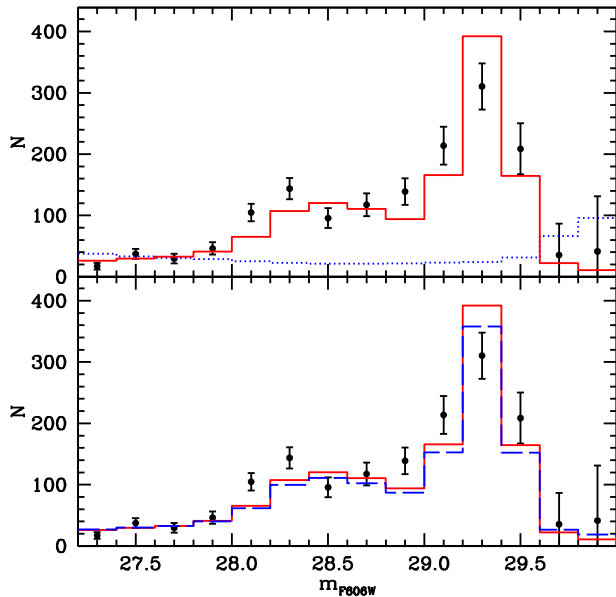


Figure 13. As Fig. 7, but showing the top panel with a solid line, the same 13 Gyr LF plotted as a solid line in Fig. 10, populated by WDs with hydrogen envelopes and progenitor mass function with $x = -2.1$. The dotted line denotes a 13 Gyr LF calculated with the same IFMR and progenitor mass function but using WD models with helium envelopes (see text for details). The lower panel shows the same LF for hydrogen envelope WDs (solid line) and the LF for a composite population made of both hydrogen and helium envelope WDs (dashed line). The number ratio of hydrogen-to-helium envelope objects at formation is set to 90:10 at birth (see text for details).

This also means that the number counts are insensitive to the choice of the exponent of the progenitor mass function.

The comparison clearly shows that the population of helium envelope objects must be a relatively small fraction of the total number of WDs observed, otherwise the shape of the cluster LF would be very different from what is observed.⁶

On the observational side, Moehler et al. (2004) have studied spectroscopically five objects along the cluster CS with T_{eff} between $\sim 20\,000$ and $\sim 13\,000$ K (corresponding to m_{F606W} between ~ 24 and 24.7 according to our isochrones) and found they all have hydrogen atmospheres. Renzini et al. (1996) found that in their sample of 21 bright cluster WDs with T_{eff} between $\sim 20\,000$ and $\sim 10\,000$ K, the positions of two objects in the CMD are consistent with helium atmosphere WDs (~ 10 per cent).

We also note that the faint-end of the observed CS in Fig. 6 has a width larger than what predicted from the artificial star analysis. This could be hinting at the presence of helium envelope WDs that towards the faint-end of the CS are expected to become increasingly redder than their hydrogen envelope counterpart, as shown in Fig. 6.

Assuming that the Renzini et al. (1996) result represents the true ratio between hydrogen and helium envelope WDs at birth (9:1), these WDs populate a magnitude range within the limits of the Chen et al. (2022) study, where the number ratio ‘slow’/‘canonical’ WDs is equal to $\sim 70/30$. Our adopted WD models show that in

the magnitude range of Chen et al. (2022) analysis, the $0.54M_{\odot}$ (a representative mass populating this temperature range) models with hydrogen and helium envelopes have the same cooling times, implying that Chen et al. (2022) results obtained considering only hydrogen envelope models are unaffected if we include this small fraction (~ 10 per cent of the total) of helium envelope objects in the 30 per cent of ‘fast’ WDs without hydrogen burning.

For our purposes, we have calculated an LF made of a composite population with 90 per cent hydrogen envelope objects, and 10 per cent helium envelope WDs at birth, both for an exponent of the progenitor mass function $x = -2.1$, assuming there is no transformation from hydrogen-to-helium envelopes during the cooling. The lower panel of Fig. 13 replaces the LF of helium envelope models with this composite population in the magnitude range of interest to our analysis. Due to the generally faster cooling times of the helium envelope models, they make only a few per cent of the total number of objects within the magnitude bins centred between $m_{\text{F606W}} = 27.3$ and 29.5 , and the composite LF is almost identical to the one calculated with just hydrogen envelope WDs.

9 SUMMARY

This is our conclusive study on the WD CS of NGC 6752, which was the main goal of the multicycle *HST* large program GO-15096/15491. With respect to Paper III, which was based only on the first half of the data, we now double the total exposure time and extended our work, taking advantage of all the data collected to better define the cluster CS and its LF. This has allowed us to perform robust and conclusive comparisons with theory; more specifically, the WD models by Salaris et al. (2022).

We found that the shape of cluster WD LF is very similar to its counterpart in the metal richer, redder HB GC M4 (Bedin et al. 2009), and that theoretical LFs for hydrogen envelope WD models calculated with the Cassisi et al. (2007) electron conduction opacities closely follow the observed trend of star counts as a function of magnitude. The observed LF peaks at $m_{\text{F606W}} \simeq 29.3 \pm 0.1$, consistent within uncertainties with what has been previously reported, and the magnitude of this peak is matched by WD isochrones with ages between 12.7 and 13.5 Gyr, consistent with the cluster age derived from the MS TO and subgiant branch. We confirm that the predicted magnitude of the LF peak and cutoff is unaffected by realistic variations of the adopted IFMR and the progenitor mass function. We also find that the impact of the cluster multiple populations on the WD LF for m_{F606W} larger than ~ 27.3 is negligible.

Our analysis also reveals a possible hint of an underestimate of the cooling time-scales of models in the magnitude range between $m_{\text{F606W}} = 28.1$ and 28.9 . However, different choices of the IFMR can reduce the discrepancy between theoretical and empirical star counts in this magnitude range to below 2σ . A hint of a similar discrepancy can be found in the analysis of the WD LF in M4 by Bedin et al. (2009).

Renzini et al. (1996) photometry of a small sample of bright cluster WDs suggests the presence of a small fraction (~ 10 per cent) of WDs with helium envelopes. We find that this fraction of helium envelope objects has a negligible impact on the shape of the LF, and is potentially responsible for the colour width of the faint end of the observed CS.

Finally, we find that hydrogen envelope models calculated with the Blouin et al. (2020) electron conduction opacities provide WD ages in disagreement (they are too low) with the TO age.

⁶This is strictly true in the hypothesis that hydrogen envelope WDs do not transform to helium envelope objects during their cooling evolution, because of convective mixing with the underlying more massive helium layers (see e.g. the discussion in Davis et al. 2009).

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DATA AVAILABILITY

The data underlying this article were accessed from the Mikulski Archive for Space Telescopes (MAST), available at <https://archive.stsci.edu/hst/search.php>. All data come from *HST* programs GO-15096 and GO-14662 (on both P.I. Bedin). The full list of observations are reported at these two URLs following the archival pages: www.stsci.edu/cgi-bin/get-proposal-info?id=15491 and www.stsci.edu/cgi-bin/get-proposal-info?id=15096 following the link HST Archive Information. The stellar models employed in this article are available at <http://basti-iac.oa-abruzzo.inaf.it/>

As supplementary online material for the present work, we also release the observed LF, the corrected LF, the completeness, the photometric errors, and the observed CMD.

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SUPPORTING INFORMATION

Supplementary data are available at *MNRAS* online.

Supplementary.tar.gz

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